the standard cubic is that:

$$\begin{cases} v^3 + v^2 + xv = y \\ 3v^2 + 2v + x = 0. \end{cases} \tag{3}$$

This is also the necessary and sufficient condition that the point (x, y) lie on the envelope of the straight line. Every isoradical straight line is therefore tangent to the locus

$$4x^3 - x^2 + 18xy + 27y^2 - 4y = 0 \tag{4}$$

obtained by eliminating v from (3).

All that is necessary, therefore, in order to solve a cubic, is to reduce it to standard form, obtaining the quantities x and y. Plot the corresponding point, and drop the three tangents to the locus (4). The slopes of the three tangents will be the three roots of the equation as written in standard form.

Note that the locus (4) can be plotted once for all, and in place of plotting a cubic equation, one need only plot a point for each problem to be solved.

QUESTIONS AND DISCUSSIONS.

EDITED BY W. A. HURWITZ, Cornell University, Ithaca, N. Y.

DISCUSSIONS.

Psychologists today are restoring to honor, albeit in much modified form, the Old Reliable Dream Book. Theologists and educators are seeking to read the future. It is not surprising that a claim should be made for the use of mathematics as a means of prediction. Such a claim is stated by Professor Weaver in the first discussion this month. His conception must not be dismissed as fantastic without an investigation into its meaning. He makes quite clear that no prediction of individual events or circumstances is intended; his hope is at most that the progress of history in the large or average sense may be forecasted. In support of his contention may be cited the service of mathematics in exactly this function of prediction in connection with the physical sciences. It may even be argued that an example of similar application to the social sciences is to be found in the generally accepted theory of the periodicity of economic cycles of prosperity and depression, with an approximate period of ten or eleven years. Against this argument may be placed the apparently hopeless complexity, as regards mathematical formulation, of the social problem, compared with the physical problem. It is doubtful whether the reduction to concise qualitative laws, or even the specification of the independent variables in terms of which such laws are to be stated lies within the mental potentialities of the human race.

Several details in Professor Weaver's account suggest comment. For example, it is not obvious that the analogy of the determination of all values of an everywhere analytic function by the values of the function and its derivatives

at a single point, is the most natural one. Would it not be more appropriate, in view of the suggestions of mathematical physics, to suppose that the history of the universe is contained in a vast system of differential equations, or in view of Volterra's hypothesis of "heredity," of integro-differential or other still more complicated equations, so that the knowledge of a *finite* set of initial conditions would suffice to determine a solution completely?

Also the notion of the "most probable" of all possible trends of natural phenomena may be much less simple than special cases would indicate. That determination of the history of a country or the world which will produce the "most probable" of all conceivable birth-rates may not coincide with the determination which will produce the "most probable" duration of life. This almost obvious comment is not meant as an objection to Professor Weaver's remarks, nor is it implied that a "most probable" configuration under the whole imaginable set of auxi iary conditions may not also exist; such questions would naturally have to be settled if the present vision should come within appreciable range of reality.

The second discussion brings us back to something more concrete. Dr. Sensenig indicates a form of derivation of the integral as the limit of a sum, which practically amounts to a proof, for the case of an analytic function, of the identity of the concepts *integral* and *anti-derivative*. It is at times useful to have such proofs at hand, even in elementary instruction, as aids in producing conviction in the minds of students. Of course, the use of the formula for Σn^m will prove an obstacle to the use of Dr. Sensenig's work for that purpose. It is doubtless universally familiar that the convergence of the Cauchy sum, in the case of a monotonically increasing or decreasing continuous function, can be cast in a geometric form so vivid as to be accepted with ease by the ordinary classes in calculus.

As the third discussion appears an alternative derivation of the expressions for the half-angles of a plane triangle in terms of the sides. Professor Baudin obtains the relations from the law of sines instead of from the law of cosines. The work is less simple than the ordinary methods; it is interesting, however, to see that it can be carried out.

I. Forecast.

By Warren Weaver, California Institute of Technology.

The foundations of science and scientific thought have been subjected in the last fifty years to examination of a most critical sort. This examination has resulted in development along two general lines; a filling of needful matter into the interstices of the even yet porous body of logical thought where the general trend of the previously accepted body has been found still tenable, and an opening up of new problems in those regions where the old theories have been found inadequate or merely approximate. As an example of the first type there comes to one's mind the rigor that has been brought to the fundamental concepts of the calculus by means of function theory. The outstanding illustration of the latter